CHARLES DARWIN UNIVERSITY

Family Name	
Given Names	
Student Number	
Teaching Period	Semester 2, 2015

FINAL EXAMINATION	DURATION
CMQ101 – Quantitative Analysis for Business	Reading Time: **10** minutes Writing Time: **180** minutes

INSTRUCTIONS TO CANDIDATES

There are three parts.

Part A is Multiple Choice, Part B is Short Answers and Part C is long answer questions.

Please answer all the parts in the same examination booklet provided. Also please clearly mark the parts you are answering.

EXAM CONDITIONS

This is a RESTRICTED OPEN BOOK examination

Any non-programmable calculator is permitted

No handwritten notes are permitted

Hard copy, unannotated English translation dictionary only

Answer on the supplied examination material/s only

ADDITIONAL AUTHORISED MATERIALS	EXAMINATION MATERIALS TO BE SUPPLIED
No additional printed material is permitted	1 x 16 Page Book Formula Sheet/s

THIS EXAMINATION IS PRINTED

DOUBLE-SIDED.

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Part B
Short Answer Questions
Total No of Marks for this section: 10 Marks

This section should be answered in the Answer Booklet provided.

Marks for each question are indicated.
Suggested Time allocation for Section B: 20 minutes

Question B1

In a 2006 poll of 600 single adults by a matchmaking website, 24% said it was OK to tell “white lies” online.

a) If a sample of size 600 yields a sample proportion of 0.24, the approximate standard deviation of the distribution of sample proportion is $\sqrt{\frac{0.24(1-0.24)}{600}} = 0.017$. What is the 98% confidence interval for the proportion of all single adults who would say it’s OK to tell “white lies” online?

(2 Marks)

b) Based on your interval, is it plausible that one fourth (25%) of all single adults would say it’s OK to tell “white lies” online? Fully state your reasons.

(2Marks)

c) Explain what would the 99% confidence interval be compared to a 98% confidence interval?

(2 Marks)

d) Explain the effect an increase in sample size would have on the interval?

(2 Marks)

Question B2

According to a March, 2005 article in the New England Journal of Medicine, "On February 20, 1905, ruling in *Jacobson v. Massachusetts*, the U.S. Supreme Court upheld the right of the city of Cambridge, Massachusetts, to mandate vaccination against smallpox." The article explains that "by 1902, vaccination was well established in Massachusetts. Nevertheless, smallpox remained a persistent visitor: [...] in 1901, there were 773 cases and 97 deaths, and in 1902, 2314 cases and 284 deaths."

The observed counts are recorded in a two-way table.

	Died	Survived	Total
1901	97	676	773
1902	284	2030	2314
Total	381	2706	3087

a) If there were no significant difference in the death rate between 1901 and 1902, how many of the 773 cases in Massachusetts in 1901 would we expect to result in death?

(1 mark)

b) How does the expected count in (a) compare to the observed count that died in 1901? Explain your answer fully.

(1mark)

Part C
Long Answer Question
Total Number of Marks for this section: 25 Marks

This section should be answered in the Answer Booklet provided.

Marks for the question are indicated.
Suggested Time allocation for Section C: 130 minutes

Question C1

An executive at a telecommunications company is interested in the relationship between an individual's income and their mobile phone usage. In particular, to help her in pricing and marketing strategies, she is interested in ascertaining whether she can use an individual's gross annual income to predict how much time they will spend on making National Direct Calls from their mobile phone per week.

She surveyed 12 mobile phone users and recorded their annual incomes and time (in minutes) spent each week making National Direct Calls. The data are presented in sequence, according to gross annual income.

Annual income ($000)	23	29	29	35	42	46	50	54	64	66	76	78
Weekly time on National Direct Calls (minutes)	69	95	102	118	126	125	138	178	156	184	176	225

(a) Develop a Regression model to predict mobile phone call times using annual income.

(10 marks)

(b) Calculate and interpret the coefficient of determination

(5 marks)

(c) Test the slope of the estimated regression to determine whether there is a significant positive relationship between mobile phone call times and annual income. Use $\alpha = 0.01$. What conclusion can you draw from this about the regression model you have developed?

(10 marks)

Formula Card for Johnson & Kuby, ELEMENTARY STATISTICS, Eleventh Edition

Sample mean:

$$\bar{x} = \frac{\sum x}{n} \quad (2.1)$$

Depth of sample median:

$d(\tilde{x}) = (n + 1)/2$ (2.2)

Range: $H - L$ (2.4)

Sample variance:

$$s^2 = \frac{\sum(x - \bar{x})^2}{n - 1} \quad (2.5)$$

or

$$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n - 1} \quad (2.9)$$

Sample standard deviation:

$s = \sqrt{s^2}$ (2.6)

Chebyshev's theorem: at least $1 - (1/k^2)$ (p. 99)

Sum of squares of x:

$SS(x) = \Sigma x^2 - ((\Sigma x)^2/n)$ (2.8)

Sum of squares of y:

$SS(y) = \Sigma y^2 - ((\Sigma y)^2/n)$ (3.3)

Sum of squares of xy:

$SS(xy) = \Sigma xy - ((\Sigma x \cdot \Sigma y)/n)$ (3.4)

Pearson's correlation coefficient:

$r = SS(xy)/\sqrt{SS(x) \cdot SS(y)}$ (3.2)

Equation for line of best fit: $\hat{y} = b_o + b_1 x$ (p. 146)

Slope for line of best fit: $b_1 = SS(xy)/SS(x)$ (3.6)

y-intercept for line of best fit:

$b_0 = [\Sigma y - (b_1 \cdot \Sigma x)]/n$ (3.7)

Empirical (observed) probability:

$P'(A) = n(A)/n$ (4.1)

Theoretical probability for equally likely sample space:

$P(A) = n(A)/n(S)$ (4.2)

Complement rule:

$P(\text{not } A) = P(\bar{A}) = 1 - P(A)$ (4.3)

General addition rule:

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ (4.4)

General multiplication rule:

$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$ (4.5)

Special addition rule for mutually exclusive events:

$P(A \text{ or } B \text{ or} \ldots \text{or } E) = P(A) + P(B) + \cdots + P(E)$ (4.6)

Special multiplication rule for independent events:

$P(A \text{ and } B \text{ and} \ldots \text{and } E) = P(A) \cdot P(B) \cdot \cdots \cdot P(E)$ (4.7)

Mean of discrete random variable:

$\mu = \Sigma[xP(x)]$ (5.1)

Variance of discrete random variable:

$\sigma^2 = \Sigma[x^2P(x)] - \{\Sigma[xP(x)]\}^2$ (5.3a)

Standard deviation of discrete random variable:

$\sigma = \sqrt{\sigma^2}$ (5.4)

Factorial: $n! = (n)(n-1)(n-2) \cdot \cdots \cdot 2 \cdot 1$ (p. 248)

Binomial coefficient:

$$\binom{n}{x} = \frac{n!}{x!(n-x)!} \quad (5.6)$$

Binomial probability function:

$$P(x) = \binom{n}{x} \cdot p^x \cdot q^{n-x}, x = 0, 1, 2, \ldots, n \quad (5.5)$$

Mean of binomial random variable: $\mu = np$ (5.7)

Standard deviation, binomial random variable:

$\sigma = \sqrt{npq}$ (5.8)

Standard score: $z = (x - \mu)/\sigma$ (6.3)

Standard score for $\bar{x}$: $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ (7.2)

Confidence interval for mean, μ (σ known):

$\bar{x} \pm z(\alpha/2) \cdot (\sigma/\sqrt{n})$ (8.1)

Sample size for $1 - \alpha$ confidence estimate for μ:

$n = [z(\alpha/2) \cdot \sigma/E]^2$ (8.3)

Calculated test statistic for H_o: $\mu = \mu_0$ (σ known):

$z\bigstar = (\bar{x} - \mu_0)/(\sigma/\sqrt{n})$ (8.4)

Confidence interval estimate for mean, μ (σ unknown):

$\bar{x} \pm t(\text{df}, \alpha/2) \cdot (s/\sqrt{n})$ with df $= n - 1$ (9.1)

Calculated test statistic for H_o: $\mu = \mu_0$ (σ unknown):

$t\bigstar = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$ with df $= n - 1$ (9.2)

Confidence interval estimate for proportion, p:

$p' \pm z(\alpha/2) \cdot \sqrt{(p'q')/n}, \quad p' = x/n$ (9.6)

Calculated test statistic for H_o: $p = p_0$:

$z\bigstar = (p' - p_0)/\sqrt{(p_0q_0/n)}, \quad p' = x/n$ (9.9)

Calculated test statistic for H_o: $\sigma^2 = \sigma_0^2$ or $\sigma = \sigma_0$:

$\chi^2\bigstar = (n-1)s^2/\sigma_0^2, \quad \text{df} = n - 1$ (9.10)

Mean difference between two dependentsamples:

Paired difference: $d = x_1 - x_2$ (10.1)

Confidence interval for mean difference, μ_d:

$\bar{d} \pm t(\text{df}, \alpha/2) \cdot s_d/\sqrt{n}$ with df$= n - 1$ (10.2)

Sample mean of paired differences:

$\bar{d} = \Sigma d/n$ (10.3)

Sample standard deviation of paired differences:

$$s_d = \sqrt{\frac{\sum d^2 - \frac{(\sum d)^2}{n}}{n-1}} \quad (10.4)$$

Calculated test statistic for H_o: $\mu_d = \mu_0$:

$t\star = (\bar{d} - \mu_0)/(s_d/\sqrt{n}), \quad \text{df} = n - 1$ (10.5)

Difference between means of two independent samples:

Degrees of freedom:

df = smaller of $(n_1 - 1)$ or $(n_2 - 1)$ (p. 496)

Confidence interval estimate for $\mu_1 - \mu_2$:

$(\bar{x}_1 - \bar{x}_2) \pm t(\text{df}, \alpha/2)\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}$ (10.8)

Calculated test statistic for H_o: $\mu_1 - \mu_2 = (\mu_1 - \mu_2)_0$:

$t\star = [(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0]/\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}$ (10.9)

Difference between proportions of two independent samples:

Confidence interval for $p_1 - p_2$:

$$(p_1' - p_2') \pm z(\alpha/2) \cdot \sqrt{\frac{p_1'q_1'}{n_1} + \frac{p_2'q_2'}{n_2}} \quad (10.11)$$

Pooled observed probability:

$p_p' = (x_1 + x_2)/(n_1 + n_2)$ (10.13)

$q_p' = 1 - p_p'$ (10.14)

Calculated test statistic for H_o: $p_1 - p_2 = 0$:

$$z\star = \frac{p_1' - p_2'}{\sqrt{(p_p')(q_p')\left[\left(\frac{1}{n_1} + \frac{1}{n_2}\right)\right]}} \quad (10.15)$$

Ratio of variances between two independent samples:

Calculated test statistic for H_o: $\sigma_1^2/\sigma_2^2 = 1$:

$F\star = s_1^2/s_2^2$ (10.16)

Calculated test statistic for enumerative data:

$\chi^2\star = \Sigma[(O - E)^2/E]$ (11.1)

Multinomial experiment:

Degrees of freedom: df $= k - 1$ (11.2)

Expected frequency: $E = n \cdot p$ (11.3)

Test for independence or Test of homogeneity:

Degrees of freedom:

df $= (r - 1) \cdot (c - 1)$ (11.4)

Expected value: $E = (R \cdot C)/n$ (11.5)

Mathematical model:

$x_{c,k} = \mu + F_c + \epsilon_{k(c)}$ (12.13)

Total sum of squares:

$$\text{SS(total)} = \Sigma(x^2) - \frac{(\Sigma x)^2}{n} \quad (12.2)$$

Sum of squares due to factor:

$$\left[\left(\frac{C_1^2}{k_1}\right) + \left(\frac{C_2^2}{k_2}\right) + \left(\frac{C_3^2}{k_3}\right) + \cdots\right] - \left[\frac{(\Sigma x)^2}{\text{n}}\right] \quad (12.3)$$

Sum of squares due to error:

$\text{SS(error)} = \Sigma(x^2) - [(C_1^2/k_1) + (C_2^2/k_2) + (C_3^2/k_3) + \cdots]$ (12.4)

Degrees of freedom for total:

df(total) $= n - 1$ (12.6)

Degrees of freedom for factor:

df(factor) $= c - 1$ (12.5)

Degrees of freedom for error:

df(error) $= n - c$ (12.7)

Mean square for factor:

MS(factor) = SS(factor)/df(factor) (12.10)

Mean square for error:

MS(error) = SS(error)/df(error) (12.11)

Calculated test statistic for H_o: Mean value is same at all levels:

$F\star$ = MS(factor)/MS(error) (12.12)

Covariance of x and y:

$\text{covar}(x, y) = \Sigma[(x - \bar{x})(y - \bar{y})]/(n - 1)$ (13.1)

Pearson's correlation coefficient:

$r = \text{covar}(x, y)/(s_x \cdot s_y)$ (13.2)

or

$r = \text{SS}(xy)/\sqrt{\text{SS}(x) \cdot \text{SS}(y)}$ (3.2) or (13.3)

Experimental error: $e = y - \hat{y}$ (13.5)

Estimated variance of error: $s_e^2 = \Sigma(y - \hat{y})^2/(n - 2)$ (13.6)

or

$$s_e^2 = \frac{(\Sigma y^2) - (b_0)(\Sigma y) - (b_1)(\Sigma xy)}{n - 2} \quad (13.8)$$

Standard deviation about the line of best fit:

$s_e = \sqrt{s_e^2}$ (13.9)

Estimate for variance of slope:

$$s_{b_1}^2 = \frac{s_e^2}{\text{SS}(x)} = \frac{s_e^2}{\Sigma x^2 - [(\Sigma x)^2/n]} \quad (13.12)$$

Confidence interval for β_1:

$b_1 \pm t(\text{df}, \alpha/2) \cdot s_{b1}$ (13.14)

Calculated test statistic for H_o: $\beta_1 = 0$:

$t\star = (b_1 - \beta_1)/s_{b1}$ with df $= n - 2$ (13.15)

Confidence interval for mean value of y at x_0:

$$\hat{y} \pm t(n - 2, \alpha/2) \cdot s_e \cdot \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{\text{SS}(x)}} \quad (13.17)$$

Prediction interval for y at x_0:

$$\hat{y} \pm t(n - 2, \alpha/2) \cdot s_e \cdot \sqrt{1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{\text{SS}(x)}} \quad (13.16)$$

Mann–Whitney U test:

$U_a = n_a \cdot n_b + [(n_b) \cdot (n_b + 1)/2] - R_b$ (14.3)

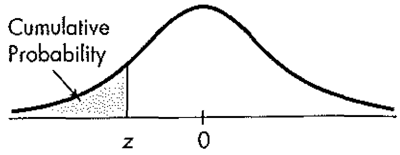
$U_b = n_a \cdot n_b + [(n_a) \cdot (n_a + 1)/2] - R_a$ (14.4)

Spearman's rank correlation coefficient:

$$r_s = 1 - \left[\frac{6\Sigma d^2}{n(n^2 - 1)}\right] \quad (14.11)$$

TABLE 3

Cumulative Areas of the Standard Normal Distribution



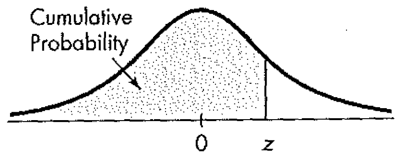
The entries in this table are the cumulative probabilities for the standard normal distribution z (that is, the normal distribution with mean 0 and standard deviation 1). The shaded area under the curve of the standard normal distribution represents the cumulative probability to the left of a z-value in the **left-hand tail.**

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−5.0	0.0000003									
−4.5	0.000003									
−4.0	0.00003	0.00003	0.00003	0.00003	0.00003	0.00003	0.00002	0.00002	0.00002	0.00002
−3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003
−3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
−3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
−3.6	0.0002	0.0002	0.0002	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
−3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0014	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0042	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0076	0.0073	0.0071	0.0070	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0126	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1094	0.1075	0.1057	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1563	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2207	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

For specific details about using this table to find probabilities, see pages 272–274, 292–294; *p*-values, pages 375–377. Table 3 was generated using Minitab.

TABLE 3

Cumulative Areas of the Standard Normal Distribution (*continued*)



The entries in this table are the cumulative probabilities for the standard normal distribution z (that is, the normal distribution with mean 0 and standard deviation 1). The shaded area under the curve of the standard normal distribution represents the cumulative probability to the left of a z-value in the **left-hand tail.**

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5754
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7258	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7518	0.7549
0.7	0.7580	0.7612	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7996	0.8023	0.8051	0.8079	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9430	0.9441
1.6	0.9452	0.9463	0.9474	0.9485	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9700	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9762	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9983	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997
4.0	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998	0.99998	0.99998	0.99998
4.5	0.999997									
5.0	0.9999997									

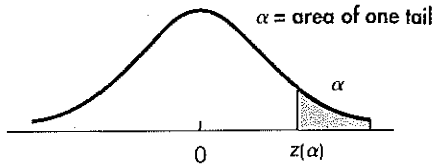
Table 3 was generated using Minitab.

TABLE 4

Critical Values of Standard Normal Distribution

A ONE-TAILED SITUATIONS

The entries in this table are the critical values for z for which the area under the curve representing α is in the right-hand tail. Critical values for the left-hand tail are found by symmetry.

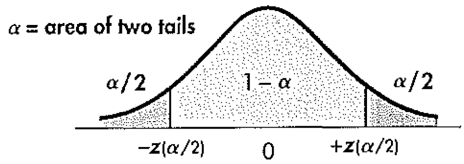


	Amount of α in one tail						
α	0.25	0.10	0.05	0.025	0.02	0.01	0.005
$z(\alpha)$	0.67	1.28	1.65	1.96	2.05	2.33	2.58

One-tailed example:
$\alpha = 0.05$
$z(\alpha) = z(0.05) = 1.65$

B TWO-TAILED SITUATIONS

The entries in this table are the critical values for z for which the area under the curve representing α is split equally between the two tails.



	Amount of α in two tails					
α	0.25	0.20	0.10	0.05	0.02	0.01
$z(\alpha/2)$	1.15	1.28	1.65	1.96	2.33	2.58
$1-\alpha$	0.75	0.80	0.90	0.95	0.98	0.99
	Area in the "center"					

Two-tailed example:
$\alpha = 0.05$ or $1 - \alpha = 0.95$
$\alpha/2 = 0.025$
$z(\alpha/2) = z(0.025) = 1.96$

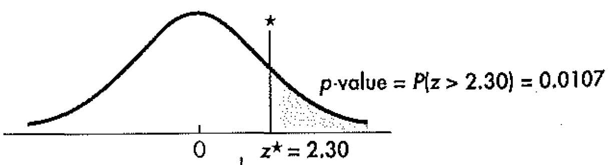
For specific details about using Table A to find critical values, see page 393.

For specific details about using Table B to find confidence coefficients, see pages 348, 350, 356; for critical values, see pages 393, 395–396.

TABLE 5

p-Values for Standard Normal Distribution

The entries in this table are the p-values related to the right-hand tail for the calculated $z\bigstar$ for the standard normal distribution.



$z\bigstar$	p-value	$z\bigstar$	p-value	$z\bigstar$	p-value	$z\bigstar$	p-value	$z\bigstar$	p-value
0.00	0.5000	0.80	0.2119	1.60	0.0548	2.40	0.0082	3.20	0.0007
0.05	0.4801	0.85	0.1977	1.65	0.0495	2.45	0.0071	3.25	0.0006
0.10	0.4602	0.90	0.1841	1.70	0.0446	2.50	0.0062	3.30	0.0005
0.15	0.4404	0.95	0.1711	1.75	0.0401	2.55	0.0054	3.35	0.0004
0.20	0.4207	1.00	0.1587	1.80	0.0359	2.60	0.0047	3.40	0.0003
0.25	0.4013	1.05	0.1469	1.85	0.0322	2.65	0.0040	3.45	0.0003
0.30	0.3821	1.10	0.1357	1.90	0.0287	2.70	0.0035	3.50	0.0002
0.35	0.3632	1.15	0.1251	1.95	0.0256	2.75	0.0030	3.55	0.0002
0.40	0.3446	1.20	0.1151	2.00	0.0228	2.80	0.0026	3.60	0.0002
0.45	0.3264	1.25	0.1056	2.05	0.0202	2.85	0.0022	3.65	0.0001
0.50	0.3085	1.30	0.0968	2.10	0.0179	2.90	0.0019	3.70	0.0001
0.55	0.2912	1.35	0.0885	2.15	0.0158	2.95	0.0016	3.75	0.0001
0.60	0.2743	1.40	0.0808	2.20	0.0139	3.00	0.0013	3.80	0.0001
0.65	0.2578	1.45	0.0735	2.25	0.0122	3.05	0.0011	3.85	0.0001
0.70	0.2420	1.50	0.0668	2.30	0.0107	3.10	0.0010	3.90	0+
0.75	0.2266	1.55	0.0606	2.35	0.0094	3.15	0.0008	3.95	0+

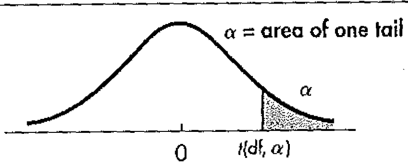
For specific details about using this table to find p-values, see pages 376–378.

TABLE 6

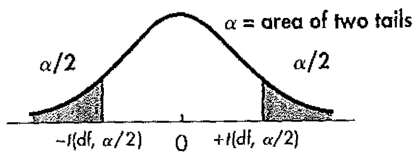
Critical Values of Student's t-Distribution

The entries in this table are the critical values of the Student's t-distribution, for which the area under the curve is: a) in the right-hand tail, or b) in two tails. See the illustrations at the bottom of the page.

Area in One Tail	0.25	0.10	0.05	0.025	0.01	0.005
Area in Two Tails: df	**0.50**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**
3	0.765	1.64	2.35	3.18	4.54	5.84
4	0.741	1.53	2.13	2.78	3.75	4.60
5	0.727	1.48	2.02	2.57	3.36	4.03
6	0.718	1.44	1.94	2.45	3.14	3.71
7	0.711	1.41	1.89	2.36	3.00	3.50
8	0.706	1.40	1.86	2.31	2.90	3.36
9	0.703	1.38	1.83	2.26	2.82	3.25
10	0.700	1.37	1.81	2.23	2.76	3.17
11	0.697	1.36	1.80	2.20	2.72	3.11
12	0.695	1.36	1.78	2.18	2.68	3.05
13	0.694	1.35	1.77	2.16	2.65	3.01
14	0.692	1.35	1.76	2.14	2.62	2.98
15	0.691	1.34	1.75	2.13	2.60	2.95
16	0.690	1.34	1.75	2.12	2.58	2.92
17	0.689	1.33	1.74	2.11	2.57	2.90
18	0.688	1.33	1.73	2.10	2.55	2.88
19	0.688	1.33	1.73	2.09	2.54	2.86
20	0.687	1.33	1.72	2.09	2.53	2.85
21	0.686	1.32	1.72	2.08	2.52	2.83
22	0.686	1.32	1.72	2.07	2.51	2.82
23	0.685	1.32	1.71	2.07	2.50	2.81
24	0.685	1.32	1.71	2.06	2.49	2.80
25	0.684	1.32	1.71	2.06	2.49	2.79
26	0.684	1.31	1.70	2.05	2.47	2.77
27	0.684	1.31	1.70	2.05	2.47	2.77
28	0.683	1.31	1.70	2.05	2.47	2.76
29	0.683	1.31	1.70	2.05	2.46	2.76
30	0.683	1.31	1.70	2.04	2.46	2.75
35	0.682	1.31	1.69	2.03	2.44	2.72
40	0.681	1.30	1.68	2.02	2.42	2.70
50	0.679	1.30	1.68	2.01	2.40	2.68
70	0.678	1.29	1.67	1.99	2.38	2.65
100	0.677	1.29	1.66	1.98	2.36	2.63
df > 100	0.675	1.28	1.65	1.96	2.33	2.58



One-tailed example:
df = 9 and $\alpha = 0.10$
$t(\text{df}, \alpha) = t(9, 0.10) = 1.38$



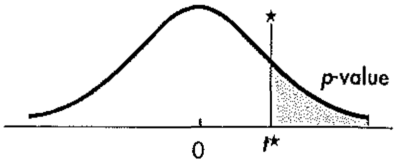
Two-tailed example:
df = 14, $\alpha = 0.02$, $1 - \alpha = 0.98$
$t(\text{df}, \alpha/2) = t(14, 0.01) = 2.62$

For specific details about using this table to find confidence coefficients, see pages 415–416, 418; p-values, pages 421–422; critical values, pages 415, 421. Table 6 was generated using Minitab.

TABLE 7

Probability-Values for Student's t-distribution

The entries in this table are the p-values related to the right-hand tail for the calculated $t\bigstar$ value for the t-distribution of df degrees of freedom.



	Degrees of Freedom														
$t\bigstar$	3	4	5	6	7	8	10	12	15	18	21	25	29	35	df ≥ 45
0.0	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.1	0.463	0.463	0.462	0.462	0.462	0.461	0.461	0.461	0.461	0.461	0.461	0.461	0.461	0.460	0.460
0.2	0.427	0.426	0.425	0.424	0.424	0.423	0.423	0.422	0.422	0.422	0.422	0.422	0.421	0.421	0.421
0.3	0.392	0.390	0.388	0.387	0.386	0.386	0.385	0.385	0.384	0.384	0.384	0.383	0.383	0.383	0.383
0.4	0.358	0.355	0.353	0.352	0.351	0.350	0.349	0.348	0.347	0.347	0.347	0.346	0.346	0.346	0.346
0.5	0.326	0.322	0.319	0.317	0.316	0.315	0.314	0.313	0.312	0.312	0.311	0.311	0.310	0.310	0.310
0.6	0.295	0.290	0.287	0.285	0.284	0.283	0.281	0.280	0.279	0.278	0.277	0.277	0.277	0.276	0.276
0.7	0.267	0.261	0.258	0.255	0.253	0.252	0.250	0.249	0.247	0.246	0.246	0.245	0.245	0.244	0.244
0.8	0.241	0.234	0.230	0.227	0.225	0.223	0.221	0.220	0.218	0.217	0.216	0.216	0.215	0.215	0.214
0.9	0.217	0.210	0.205	0.201	0.199	0.197	0.195	0.193	0.191	0.190	0.189	0.188	0.188	0.187	0.186
1.0	0.196	0.187	0.182	0.178	0.175	0.173	0.170	0.169	0.167	0.165	0.164	0.163	0.163	0.162	0.161
1.1	0.176	0.167	0.161	0.157	0.154	0.152	0.149	0.146	0.144	0.143	0.142	0.141	0.140	0.139	0.139
1.2	0.158	0.148	0.142	0.138	0.135	0.132	0.129	0.127	0.124	0.123	0.122	0.121	0.120	0.119	0.118
1.3	0.142	0.132	0.125	0.121	0.117	0.115	0.111	0.109	0.107	0.105	0.104	0.103	0.102	0.101	0.100
1.4	0.128	0.117	0.110	0.106	0.102	0.100	0.096	0.093	0.091	0.089	0.088	0.087	0.086	0.085	0.084
1.5	0.115	0.104	0.097	0.092	0.089	0.086	0.082	0.080	0.077	0.075	0.074	0.073	0.072	0.071	0.070
1.6	0.104	0.092	0.085	0.080	0.077	0.074	0.070	0.068	0.065	0.064	0.062	0.061	0.060	0.059	0.058
1.7	0.094	0.082	0.075	0.070	0.066	0.064	0.060	0.057	0.055	0.053	0.052	0.051	0.050	0.049	0.048
1.8	0.085	0.073	0.066	0.061	0.057	0.055	0.051	0.049	0.046	0.044	0.043	0.042	0.041	0.040	0.039
1.9	0.077	0.065	0.058	0.053	0.050	0.047	0.043	0.041	0.038	0.037	0.036	0.035	0.034	0.033	0.032
2.0	0.070	0.058	0.051	0.046	0.043	0.040	0.037	0.034	0.032	0.030	0.029	0.028	0.027	0.027	0.026
2.1	0.063	0.052	0.045	0.040	0.037	0.034	0.031	0.029	0.027	0.025	0.024	0.023	0.022	0.022	0.021
2.2	0.058	0.046	0.040	0.035	0.032	0.029	0.026	0.024	0.022	0.021	0.020	0.019	0.018	0.017	0.016
2.3	0.052	0.041	0.035	0.031	0.027	0.025	0.022	0.020	0.018	0.017	0.016	0.015	0.014	0.014	0.013
2.4	0.048	0.037	0.031	0.027	0.024	0.022	0.019	0.017	0.015	0.014	0.013	0.012	0.012	0.011	0.010
2.5	0.044	0.033	0.027	0.023	0.020	0.018	0.016	0.014	0.012	0.011	0.010	0.010	0.009	0.009	0.008
2.6	0.040	0.030	0.024	0.020	0.018	0.016	0.013	0.012	0.010	0.009	0.008	0.008	0.007	0.007	0.006
2.7	0.037	0.027	0.021	0.018	0.015	0.014	0.011	0.010	0.008	0.007	0.007	0.006	0.006	0.005	0.005
2.8	0.034	0.024	0.019	0.016	0.013	0.012	0.009	0.008	0.007	0.006	0.005	0.005	0.005	0.004	0.004
2.9	0.031	0.022	0.017	0.014	0.011	0.010	0.008	0.007	0.005	0.005	0.004	0.004	0.004	0.003	0.003
3.0	0.029	0.020	0.015	0.012	0.010	0.009	0.007	0.006	0.004	0.004	0.003	0.003	0.003	0.002	0.002
3.1	0.027	0.018	0.013	0.011	0.009	0.007	0.006	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002
3.2	0.025	0.016	0.012	0.009	0.008	0.006	0.005	0.004	0.003	0.002	0.002	0.002	0.002	0.001	0.001
3.3	0.023	0.015	0.011	0.008	0.007	0.005	0.004	0.003	0.002	0.002	0.002	0.001	0.001	0.001	0.001
3.4	0.021	0.014	0.010	0.007	0.006	0.005	0.003	0.003	0.002	0.002	0.001	0.001	0.001	0.001	0.001
3.5	0.020	0.012	0.009	0.006	0.005	0.004	0.003	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001
3.6	0.018	0.011	0.008	0.006	0:004	0.004	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0+	0+
3.7	0.017	0.010	0.007	0.005	0.004	0.003	0.002	0.002	0.001	0.001	0.001	0.001	0+	0+	0+
3.8	0.016	0.010	0.006	0.004	0.003	0.003	0.002	0.001	0.001	0.001	0.001	0+	0+	0+	0+
3.9	0.015	0.009	0.006	0.004	0.003	0.002	0.001	0.001	0.001	0.001	0+	0+	0+	0+	0+
4.0	0.014	0.008	0.005	0.004	0.003	0.002	0.001	0.001	0.001	0+	0+	0+	0+	0+	0+

For specific details about using this table to find p-values, see pages 421–422.